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Stable non-BPS membranes on M(atrix) orientifold*

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Abstract: Examples of stable, non-BPS M-theory membrane configuration are constructed via M(atrix) theory. The stable membranes are localized on O4 or O8 orientifolds, which project Chan-Paton gauge bundle of M(atrix) zero-brane partons to USp-type. The examples are shown to be consistent with predictions based on K-theory analysis.

Keywords: M(atrix) Theories, M-Theory, D-branes.

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Contents

1. Introduction

At present, M(atrix) theory [1] is the only known nonperturbative, partonic definition of M-theory, which unifies all perturbative superstring theories. For example, it has been found that the M(atrix) theory captures successfully the dynamics of supergraviton, compactification on lower-dimensional space with or without orbifolds/orientifolds, and identification of twisted states thereof. In particular, the theory were able to encompass all known BPS states in string and M-theories.

In this paper, we point out that M(atrix) theory is also able to encompass *stable but non-BPS states* as well. We will be illustrating this in the simplest context, namely, membrane (M2) in the presence of $\mathbb{Z}_2$ orientifold planes, especially, for configurations in which they are parallel each other. In this case, two types of orientifold $O^{\pm}$ are possible, projecting onto SO or USp Chan-Paton gauge bundles [1] for the zero-brane partons.

In M(atrix) theory, the M2 on an $\mathbb{Z}_2$ orientifold is described in covering space as a pair of M2 and $\overline{\text{M2}}$ configuration, thus breaking all the supersymmetries completely (unless the membrane is on top of the orientifold). This is signalled, among others, by the presence of a tachyonic mode in the fluctuation spectrum around the M2 and $\overline{\text{M2}}$ configuration. Among the fluctuations, only those compatible with the orientifold will survive. We will thus investigate under what orientifold choices the tachyonic mode might be removed out of the fluctuation spectrum. By exploiting projection conditions of the Chan-Paton indices, we will be showing that the M2 on an $O4^+$ or $O8^+$ orientifold is a stable, non-BPS configuration.

Recently, there has been progress in classifying stable non-BPS states in string theory [2]-[11]. Generic non-BPS states are afflicted with tachyon modes, which leads to an instability for the states to decay or annihilate. As shown affirmatively by Sen [2], the tachyon condensation (with or without orientifolds) leads to plethora of BPS and (stable) non-BPS D-branes, thus shed new light on the identification of D-brane charges. Moreover, Witten [6] has shown that possible stable D-brane charge configuration formed out of tachyon condensation is most suitably classified in terms of K-theory. For example, Type I theory is equipped with $O9^-$ and it allows non-BPS D(-1), D0, D7 and D8-branes which can carry $\mathbb{Z}_2$ charges, in addition to the well-known supersymmetric D1, D5 and D9 branes. It has been also found that non-BPS D3 and D4 branes exist as localized states in $O5^-$ plane [8, 9]. D3-branes correspond to (unwrapped) D2-branes in Type IIA string theory and, due to Bott periodicity, $O9^+$ will also allow non-BPS D3-branes. From these results, one would then anticipate existence of a stable, non-BPS M2 configuration located on $O4^+$ or $O8^+$ orientifold with USp Chan-Paton gauge bundle. The present work may then be regarded as a natural extension of these results within string theory to M-theory defined via M(atrix) theory.

This paper is organized as follows. In section 2, we will briefly recapitulate the dynamics of a pair of M2-$\overline{\text{M2}}$ in M(atrix) theory. We will extend the method developed by Aharony and Berkooz [12], from which the presence of tachyon mode and orientifold projection thereof can be formulated in the most transparent way. In section 3, we will recapitulate the definition of the orientifold in M(atrix) theory and the rule of orientifold projection to fluctuation spectrum. In section 4, we will

[1] In this paper, we will be denoting orientifold p-planes with SO (USp) Chan-Paton gauge bundles as Op^- (Op^+) planes.

study M2 on O8$^\pm$ orientifold with particular attention to the fate of tachyon mode and, using the orientifold projection rules presented in section 3, show that O8$^+$ orientifold projects out the tachyon mode and hence lead to a stable, non-BPS state. In section 5, we will study M2 on O4$^\pm$ orientifold and draw the same conclusion that O4$^+$ orientifold projects out the tachyon mode. In section 6, we will conclude with a brief discussion on comparison with K-theoretic analysis, where the M2 on O4$^+$ or O8$^+$ orientifolds is known to be a stable, non-BPS state with $\mathbb{Z}_2$ charges.

2. M2-$\overline{\text{M2}}$ dynamics

We will begin with, in M(atrix) theory, dynamics of M2-$\overline{\text{M2}}$ configuration drawing particular attention to the tachyonic instability. In fact, the dynamics has been studied by Aharony and Berkooz [12]. In this section,we will repeat essential part of their analysis relevant for foregoing discussions, but in a more transparent notation. The M(atrix) theory is a nonperturbative definition of the M-theory, whose action is given by

$$S = \int dt \,\mathrm{Tr}\left[\frac{1}{2}(D_t X^I)^2 + \frac{1}{4}[X^I, X^J]^2 + \Theta^T D_t \Theta + i\Theta^T \Gamma_I [X^I, \Theta]\right]. \tag{2.1}$$

Here, X^I, Θ^a are adjoint representations of gauge group $U(N)$, and $D_t = \partial_t - i[A_0, \quad]$ is gauge covariant derivative. In M(atrix) theory, a configuration of M2-$\overline{\text{M2}}$, separated by a distance r along 9-th direction, is described by

$$X^{\mathrm{I}}{}_{\mathrm{M}\overline{\mathrm{M}}}: \quad X^1 = \begin{pmatrix} +Q & 0 \\ 0 & +\overline{Q} \end{pmatrix}, \quad X^2 = \begin{pmatrix} +P & 0 \\ 0 & -\overline{P} \end{pmatrix}, \quad X^9 = \frac{1}{2}\begin{pmatrix} +r & 0 \\ 0 & -r \end{pmatrix}, \tag{2.2}$$

and all other X^I's and Θ^a's vanishing. Here, $Q, \bar{Q}, P, \bar{P}$ are (N $\times$ N) submatrices obeying $[Q, P] = [\bar{Q}, \bar{P}] = ic\mathbf{1}$, $c = \mathcal{O}(1/N)$. It is straightforward to check that eq. (2.2) solves the equations of motion derived from the action, eq. (2.1).

As eq. (2.2) breaks all supersymmetries spontaneously, there will be corrections to the total energy of M2-$\overline{\text{M2}}$ configuration. At leading order, the correction gives rise to a static, inter-brane potential, which can be extracted by applying the Born-Oppenheimer approximation — integrate out small fluctuations around the configuration, eq. (2.2). For the Born-Oppenheimer approximation, relevant parts of fluctuations are from off-diagonal submatrices:

$$X^I = X^{\mathrm{I}}_{\mathrm{M}\overline{\mathrm{M}}} + \begin{pmatrix} 0 & Y^{\mathrm{I}} \\ Y^{\mathrm{I}\dagger} & 0 \end{pmatrix}, \qquad \Theta^a = \begin{pmatrix} 0 & \theta^a \\ \theta^{\mathrm{T}a} & 0 \end{pmatrix}. \tag{2.3}$$

Expanding the potential energy parts in eq. (2.1) up to quadratic order in $Z \equiv (Y^I, \theta^a)$, one obtains various terms of the form $\mathrm{Tr}(Z_A^\dagger \mathcal{O}_1 Z_B \mathcal{O}_2)$, where $\mathcal{O}_{1,2}$ are generic functions of classical configuration, eq. (2.2), viz. of $Q, \overline{Q}, P, \overline{P}$ and r. Taking adjoint basis for the fluctuations, $Z_{A,B}$'s, one can represent these terms compactly as

$$\begin{aligned}\mathrm{Tr}(Z_A^\dagger \mathcal{O}_1 Z_B \mathcal{O}_2) &= Z^*_{Aij}\mathcal{O}_{1im}\mathcal{O}_{2nj}Z_{Bmn}\\ &\equiv Z^*_A(\mathcal{O}_1\otimes\mathbf{1}+\mathbf{1}\otimes\mathcal{O}_2^T)Z_B\,.\end{aligned}$$

Among the quadratic terms, the bosonic fluctuations $Y_3,\ldots,Y_8$ are in a diagonal form:

$$-\sum_{\mathrm{I}=3}^{8} Y_{\mathrm{I}}^\dagger\left(\mathcal{Q}^2+\mathcal{P}^2+r^2\right)Y_{\mathrm{I}}\,, \tag{2.4}$$

where

$$\begin{aligned}\mathcal{Q} &= Q\otimes\mathbf{1}-\mathbf{1}\otimes\overline{Q}^{\mathrm{T}}\,,\\ \mathcal{P} &= P\otimes\mathbf{1}+\mathbf{1}\otimes\overline{P}^{\mathrm{T}}\,.\end{aligned} \tag{2.5}$$

Consistent with $[Q,P]=[\overline{Q},\overline{P}]=ic\mathbf{1}$, it is always possible to take a realization taking $Q,\overline{Q}$ as symmetric matrices and $P,\overline{P}$ as antisymmetric ones, so that

$$\begin{aligned}\mathcal{Q} &= Q\otimes\mathbf{1}-\mathbf{1}\otimes\overline{Q}\,,\\ \mathcal{P} &= P\otimes\mathbf{1}-\mathbf{1}\otimes\overline{P}\,.\end{aligned} \tag{2.6}$$

One thus finds that eq. (2.4) can be interpreted as a quantum mechanical system of two-particles connected by a spring, where coordinates and conjugate momenta operators of the two-particles are $Q,\overline{Q}$ and $P,\overline{P}$, respectively. Moreover, the fluctuation matrices Y_{mn}'s are interpreted as 'wave functions', where Q,P's and $\overline{Q},\overline{P}$'s act on the Chan-Paton index m and n, respectively. Thus, Y_{mn}'s are two-body wave functions in which the first and the second particles are in m-th and n-th excited states. Moreover, eq. (2.6) exhibits clearly that degrees of freedom associated with the center of mass of the analog system consisting of two particles decouple completely, as physically ought to be the case.

Using the commutation relation, $[\mathcal{Q},\mathcal{P}]=2ic\mathbf{1}$, one obtains the fluctuation spectrum of $Y_3,\ldots,Y_8$ as

$$M_\perp^2(n;r)=r^2+2c(2n+1)\,,\qquad (n=0,1,2,\ldots)\,. \tag{2.7}$$

For the bosonic fluctuations $Y_{1,2,9}$, the quadratic part is coupled one another:

$$-(Y_1^*,Y_2^*,Y_9^*)\begin{pmatrix}\mathcal{P}^2+r^2 & -\mathcal{P}\mathcal{Q}+2ic & r\mathcal{Q}\\ -\mathcal{Q}\mathcal{P}-2ic & \mathcal{Q}^2+r^2 & r\mathcal{P}\\ r\mathcal{Q} & r\mathcal{P} & \mathcal{Q}^2+\mathcal{P}^2\end{pmatrix}\begin{pmatrix}Y_1\\ Y_2\\ Y_9\end{pmatrix}. \tag{2.8}$$

Diagonalizing eq. (2.8), one obtains, in addition to a zero-mode, two fluctuation spectrum

$$\begin{aligned}M_{\|-}^2(n;r) &= r^2+2c(2n-1)\\ M_{\|+}^2(n;r) &= r^2+2c(2n+3)\,,\qquad (n=0,1,2,\cdots)\,.\end{aligned} \tag{2.9}$$

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An important point to be noted for later discussion is that, upon diagonalizing the matrix, eq. (2.8), the n-th harmonic oscillator eigenstate of Y_9 is always accompanied by a linear combination of the $(n+1)$-th and the $(n-1)$-th eigenstates of $Y_{1,2}$'s. Thus, Y_1, Y_2 are even functions when Y_9 is an odd one, and vice versa.

For fermionic coordinates, the same interpretation as the above goes through (except that the 'wave functions' θ^a are grassmannian) and the fluctuation spectrum is grouped into a pair, a consequence following from the fact that Θ^a's are projected by $\Gamma_\perp = \gamma_1\gamma_2$ for each membrane configuration. Half of them are

$$M^2_{\mathrm{F}-}(n;r) = r^2 + 2c(2n)\,, \qquad (n = 0, 1, 2, \ldots) \tag{2.10}$$

and the others are

$$M^2_{\mathrm{F}+}(n;r) = r^2 + 2c(2n+2)\,, \qquad (n = 0, 1, 2, \ldots) \tag{2.11}$$

respectively.

Now, the potential between membrane and anti-membrane arises from summing over zero point fluctuation energies, viz. graded sum of the bosonic and fermionic mass spectra obtained above:

$$\begin{aligned} V_{\mathrm{M}\overline{\mathrm{M}}}(r) &= \frac{1}{2}\mathrm{STr}M \\ &= \sum_{m=1}^{\infty}\Big(\sqrt{r^2+2c(2m-3)} + \sqrt{r^2+2c(2m+1)} - \\ &\qquad + 6\sqrt{r^2+2c(2m-1)} - 4\sqrt{r^2+2c(2m-2)} - \\ &\qquad -4\sqrt{r^2+2c(2m)}\Big)\,. \end{aligned} \tag{2.12}$$

This yields the static interaction potential between M2 and $\overline{\mathrm{M2}}$. Evidently, when the M2 and $\overline{\mathrm{M2}}$ approaches close, viz. $r < \sqrt{2c}$, the interaction develops a complex-valued potential energy, a signal of tachyonic instability. Note that the tachyon mode arises from the lowest eigenvalue of Y_1, Y_2 fluctuation, see eq. (2.9).

3. M(atrix) orientifolds

In M(atrix) theory, orientifolds are defined as quotient condition on X^{I}, Θ^a, the moduli space variables parametrizing transverse spacetime in the discrete light-cone description. To be specific, we will be considering a spacetime of the form $\mathbf{R}^{p,1} \times \mathcal{M}^{9-p}/\Gamma$, where M^{9-p} is a smooth manifold and $\Gamma = \mathbb{Z}_2$ is a discrete symmetry group acting on $\mathcal{M}^{9-p}$.

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3.1 M(atrix) $\mathbb{Z}_2$ orientifolds

The M(atrix) orientifold Op is defined as a quotient condition on covering space variables of Chan-Paton gauge bundle U($2N$):

$$\begin{aligned} X_\parallel &= +M X_\parallel{}^{\mathrm{T}} M^{-1} \qquad (\mathbf{R}^{p,1} \text{ coordinates}) \\ X_\perp &= -M X_\perp{}^{\mathrm{T}} M^{-1} \qquad (\mathcal{M}^{9-p} \text{ coordinates}) \\ \Theta_a &= \Gamma_\Omega\, M \Theta_a{}^{\mathrm{T}} M^{-1}\,. \end{aligned} \tag{3.1}$$

Here, $\Gamma_\Omega = \gamma_{9-p}\gamma_{10-p}\cdots\gamma_9$ is the product of Dirac gamma matrices of $\mathcal{M}^{9-p}$. Consistency of the quotient condition restricts possible types of the matrix M [14]. One then obtains, for a symmetric choice of $M = \mathbf{1}\otimes\sigma_1$, the projected Chan-Paton bundle is SO($2N$), while, for an antisymmetric choice of $M = \mathbf{1}\otimes\sigma_2$, the projected bundle is USp($2N$). In what follows, we will denote M(atrix) orientifold with SO or USp projections as Op^- or Op^+, respectively. The variables $X_\perp$ and half of Θ^a's belong to adjoint representation of the respective Chan-Paton gauge bundle. The variables $X_\parallel$ and the other half of Θ^a's transform as symmetric (antisymmetric) tensor representation under the SO (USp) gauge bundles, respectively. Note that, for $\mathbf{S}^1/\mathbb{Z}_2$, $\mathbf{T}^5/\mathbb{Z}_2$ and $\mathbf{T}^9/\mathbb{Z}_2$ compactifications, a consistent choice of the orientifold projection was found to be O8$^-$ [13, 14, 15], O4$^+$ [16, 17], and O0$^-$ [18, 19], respectively.[2]

3.2 Rules of orientifold projection

Consider, near an orientifold (of transverse distance $r/2$ away), placing an M2 parallel to the orientifold plane. In covering space, the configuration corresponds to an M2-$\overline{\text{M2}}$ pair, separated by a distance r.

After orientifolding, the off-diagonal submatrices of X^{I}, Θ^a's in eq. (3.1) will become symmetric or antisymmetric, depending on the choice of Chan-Paton gauge bundles. This is because, in covering space description, fluctuation modes of X^{I}, Θ^a's that are not consistent with orientifold condition will be projected out [14]. For Op^-, it turns out $Y_\parallel$'s are symmetric, while $Y_\perp$'s are antisymmetric. For Op^+, it is the opposite, viz. $Y_\parallel$'s are antisymmetric, whlie $Y_\perp$'s are symmetric. For Θ^a's, $\pm$ eigenstates of Γ_Ω are symmetric and antisymmetric, respectively.

In analyzing the fluctuation spectrum, we have interpreted Y^{I}, θ^a's as quantum-mechanical 'wave functions' of an analog sytem, which consists of two particles connected by a spring. Orientifold operation takes a transpose of each matrices, see eq. (3.1). As the Chan-Paton indices Y_{mn}, θ_{mn} are interpreted as that the first harmonic oscillator is in m-th excited state and the second in n-th excited state, under orientifold operation, the sign of relative coordinates between the two harmonic os-

[2]Our classification of M(atrix) orientifolds is slightly different from those used, for example, in [6, 8, 9]. In our notation, the Chan-Paton gauge bundles refer to M(atrix) zero-brane partons on orientifolds. In M(atrix) theory, this notation provides a more convenient bookeeping.

cillators gets reversed. This implies that, in symmetric submatrices, only even modes ($n = 0, 2, 4, \ldots$) will survive, while, in antisymmetric submatrices, only odd modes ($n = 1, 3, 5, \ldots$) will do so.

Thus, projection rules of M(atrix) orientifold would be that, for Op^- orientifold corresponding to SO Chan-Paton gauge bundle, even modes of $Y_\parallel$ and odd modes of $Y_\perp$ will only survive. For θ^a's, half of even modes and half of odd modes will survive. For Op^+ orientifold corresponding to USp Chan-Paton gauge bundle, odd modes of $Y_\parallel$ and even modes of $Y_\perp$ will only survive. For θ^a's, again, half of even modes and half of odd modes will survive.

Using orientifold projection rules stated as above, we will now analyze M2 on O8 and M2 on O4 configurations in detail.

4. M2 on M(atrix) O8-orientifold

Utilizing the fluctuation spectrum of M2-$\overline{\mathrm{M2}}$ analyzed in section 2, we will now examine stability of M2 located near an O8-orientifold in M(atrix) theory.

4.1 M2 configuration near O8-orientifold

We will be taking O8-orientifold spans $\mathbf{R}^{8,1} = (t, X^1, \ldots, X^8)$. The M2 is then located at a distance $r/2$ along X^9, the coordinate of $\mathcal{M}/\mathbb{Z}_2$, and extended along (X^1, X^2) directions. From the supersymmetry transformation rules, one finds easily that the M2-O8 configuration breaks all the supersymmetries of the M(atrix) theory.

We will now analyze the stability for O8$^\pm$-orientifold projections explicitly. This amounts to examining fate of the (complex-valued) tachyon mode, the diagonal linear combination of Y^1, Y^2 in eq. (2.8), under the orientifold projection.

4.2 M2 on O8$^-$ orientifold

For O8$^-$ orientifold corresponding to SO Chan-Paton gauge bundle, from the above projection rules, one only keeps even modes of eq. (2.7) and eq. (2.8), and even and odd modes separately for eqs. (2.10), (2.11).

Stability of the M2 configuration may be examined, for example, from the static potential when the M2 is located off the O8$^-$ orientifold at a distance $r/2$:

$$V_{\mathrm{SO}}(r) = \sum_{m=1}^{\infty} \Big(\sqrt{r^2 + 2c(4m-5)} + \sqrt{r^2 + 2c(4m-1)} + 6\sqrt{r^2 + 2c(4m-3)} - \\ -2\sqrt{r^2 + 2c(4m-4)} - 4\sqrt{r^2 + 2c(4m-2)} - 2\sqrt{r^2 + 2c(4m)} \Big) \, . \tag{4.1}$$

Terms in the first line are contribution of even modes of Y^1, Y^2, which will automatically projects into odd modes of Y^9 at the same time, and of even modes of $Y^3, \ldots, Y^8$. Likewise, those in the second line are contributions from even and odd modes of $(1 \pm \Gamma_\Omega)\Theta^a$, respectively.

At short distance, $r \to 0$, the potential, eq. (4.1), is complex-valued, as seen from the $m = 0$ contribution of the first term in the summand. This signals a tachyonic instability of the M2 located near the $O8^-$-orientifold. To explore $r \to \infty$ long distance behavior, it is convenient to reexpress the potential, eq. (4.1), into an integral representation, using the identity:

$$\sqrt{A} = -\frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{ds}{s^{3/2}} e^{-As} . \tag{4.2}$$

One finds

$$V_{\rm SO}(r) = 4r - \frac{4}{\sqrt{\pi}} \int \frac{ds}{s^{3/2}} e^{-r^2 s} e^{2cs} \frac{\sinh^4(cs)}{\sinh(4cs)} . \tag{4.3}$$

Expanding the integrand for small s, which is a convergent expansion in the limit under consideration, one obtains a long-distance behavior of the static potential for the M2-$O8^-$ orientifold configuration:

$$V_{\rm SO}(r) = +4r - \frac{3c^3}{4r^5} + \mathcal{O}\left(\frac{1}{r^7}\right), \qquad (r \to \infty) . \tag{4.4}$$

The potential is attractive. We thus conclude that, near an $O8^-$ orientifold, the M2 is attracted to the orientifold plane and eventually becomes a tachyonic configuration.

4.3 M2 on $O8^+$ orientifold

We next turn to $O8^+$ orientifold corresponding to USp Chan-Paton gauge bundle. According to the projection rules, one now need to keep odd modes of eq. (2.7) and eq. (2.9), and even and odd modes of eqs. (2.10), (2.11), respectively.

Again, we will infer the stability of M2 configuration conveniently by locating it at a distance $r/2$ off the orientifold and measure the static potential:

$$\begin{aligned} V_{\rm USp}(r) = \sum_{m=1}^{\infty} \Big(& \sqrt{r^2 + 2c(4m-3)} + \sqrt{r^2 + 2c(4m+1)} + 6\sqrt{r^2 + 2c(4m-1)} - \\ & -2\sqrt{r^2 + 2c(4m)} - 4\sqrt{r^2 + 2c(4m-2)} - 2\sqrt{r^2 + 2c(4m-4)} \Big) . \end{aligned} \tag{4.5}$$

Most significantly, one notes that the tachyon mode of eq. (2.9) (the $n = 0$ mode of $M^2_{\|-}$) is completely projected out in the background of $O8^+$-orientifold. The static potential of M2 is then well-defined for $r \to 0$.

At long distance, $r \to \infty$, using the integral representation, one finds:

$$V_{\rm USp}(r) = -4r - \frac{4}{\sqrt{\pi}} \int_0^\infty \frac{ds}{s^{3/2}} e^{-r^2 s} e^{-2cs} \frac{\sinh^4(cs)}{\sinh(4cs)} . \tag{4.6}$$

Expanding the integran for small s, one obtains:

$$V_{\rm USp}(r) = -4r - \frac{3c^3}{4r^5} + \mathcal{O}\left(\frac{1}{r^7}\right), \tag{4.7}$$

indicating a repulsive long-range force. Thus, we conclude that the M2 near an $O8^+$-orientifold exhibits a stable but non-BPS configuration.

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4.4 Compact $\mathbf{M}^1/\mathbb{Z}_2$ orientifolds and twisted sector states

In our discussion so far, we have implicitly assumed that the covering space $\mathcal{M}$ of the orientifold is noncompact. If $\mathcal{M}$ is compact, say, $\mathcal{M} = \mathbf{S}_1/\mathbb{Z}_2$, then in order to cancel anomalous fluxes carried by the orientifolds, one needs to introduce a twisted sector in the M(atrix) theory. For $\mathbf{S}_1/\mathbb{Z}_2, \mathbf{T}_5/\mathbb{Z}_2$ and $\mathbf{T}_9/\mathbb{Z}_2$ cases, complete spectrum of the twisted sector has been determined previously [14, 16, 18]. One may inquire if the conclusion in the previous subsections on non-BPS M2 on M(atrix) orientifolds would be modified in case the covering space $\mathcal{M}$ is compact, viz. if the effects of twisted sector spectrum is included.

For $\mathbf{S}_1/\mathbb{Z}_2$, consistency condition of anomalous flux and gauge anomaly cancellations fixes uniquely that the two orientifolds (located at two diagonal points on the $\mathbf{S}_1$ covering space) are $\mathrm{O8}^-$ ones and that the twisted sector consists of eight supersymmetry singlet fermions on each orientifold. The twisted sector fermions, which transform as fundamental representation under the SO(2N) Chan-Paton gauge bundle and couples only to X^9 minimally. Inferring X^9 from eq. (2.2), for *all* twisted sector fermions,[3] fluctuation energy spectrum will then be $r/2$, independent of the analog 'harmonic oscillator' excitation level n.

The off-diagonal Y^{I}, θ^a's submatrices in the untwisted sector has $(N \times N)$ degrees of freedom overall. However, for each fluctuation mode, there are N-fold degeneracy, corresponding to the center of mass motion of the analog 'two-particle harmonic oscillator' system. Projecting these degeneracy out, fluctuation energy spectrum of Y^{I}, θ^a's have effectively N-fold degeneracy only. The twisted sector fermions belong to fundamental representation of the Chan-Paton gauge bundle, so their fluctuation energy spectrum is N-fold degenerate, the same as that of untwisted sector. This implies that the leading-order, linear potential for M2 on $\mathrm{O8}^-$, eq. (4.3), is cancelled exactly by the contribution of twisted sector fermions, resulting in an attractive long-range potential.

Another possibility is that the two orientifolds at $\mathbf{S}_1/\mathbb{Z}_2$ fixed points are $\mathrm{O8}^-$ and $\mathrm{O8}^+$ type ones, respectively. In this case, as the anomalous flux from each orientifold cancels each other, there is no need to introduce a twisted sector. In this case, M2 appears a stable state locally near $\mathrm{O8}^+$ orientifold, but as it is transported near to $\mathrm{O8}^-$ orientifold, M2 ought to exhibit tachyonic instability. Thus, in this case, stability of M2 would depend, in a complicated way, both on the volume of $\mathbf{S}_1/\mathbb{Z}_2$ (viz. distance between $\mathrm{O8}^+$ and $\mathrm{O8}^-$) and on location $r/2$ of the M2.

4.5 Summary

M2 is a stable, non-BPS configuration when located on $\mathrm{O8}^+$ orientifold, but exhibits tachyonic instability when located on $\mathrm{O8}^-$ orientifold.

[3] If each twisted sector fermion is displaced at locations m_i, the fluctuation spectrum is changed to $\{r/2 + m_i\}$ accordingly.

5. M2 on M(atrix) O4-orientifold

In this section, we will extend the analysis of section 3 to M2 located on O4-orientifold in M(atrix) theory.

5.1 M2 versus O4-orientifold

We will consider an M2 extended parallel to an O4-orientifold. The O4-orientifold spans $\mathbf{R}^{4,1} = (t, X^1, \ldots, X^4)$ and an M2 stretched along (X^1, X^2) directions at a distance $r/2$ along X^9 direction. Thus, in the covering space viewpoint, the configuration is given as in eq. (2.2), viz. a M2-$\overline{\mathrm{M2}}$ dipole separated each other by a distance r. Again, the M2-O4 configuration is not a BPS state as it breaks all of thirty-two supersymmetries of the M(atrix) theory.

We will now analyze stability of the M2 on O4$^\pm$ orientifold explicitly following the same analysis as the O8 orientifold case.

5.2 M2 on O4$^-$ orientifold

For O4$^-$ orientifold corresponding to SO Chan-Paton gauge bundle of zero-brane partons, from the orientifold projection rules, one only keeps even modes of $Y^1, \ldots, Y^4$, odd modes of $Y^5, \ldots, Y^9$, and even / odd modes for each chiral sector of θ^a's.

The potential for M2-O4$^-$ is then readily obtained by summing over the modes that are left after the orientifold projection:

$$\begin{aligned} V_{\mathrm{SO}}(r) = \sum_{m=1}^{\infty} \Big(& \Big\{ \sqrt{r^2 + 2c(4m-5)} + \sqrt{r^2 + 2c(4m-1)} \Big\} + \\ & + \Big\{ 2\sqrt{r^2 + 2c(4m-3)} + 4\sqrt{r^2 + 2c(4m-1)} \Big\} - \\ & - 2\sqrt{r^2 + 2c(4m)} - 4\sqrt{r^2 + 2c(4m-2)} - 2\sqrt{r^2 + 2c(4m-4)} \Big) . \end{aligned} \tag{5.1}$$

Terms in each line come from projection to even modes of Y^1, Y^2 for the first line, which automatically picks up odd modes of Y^9, and to even and odd modes of Y^3, Y^4 and of $Y^5, \ldots, Y^8$ respectively for the second line. Terms in the third line are from even and odd modes of for $(1 \pm \Gamma_\Omega)\Theta^a$, whose decomposition follows from the fact that $[\Gamma_\perp, \Gamma_\Omega] = 0$.

Obviously, for $r \to 0$, the static potential is complex-valued, arising from the presence of a tachyon mode even after the orientifold projection (the $n = 0$ term in eq. (2.9)). Thus, M2 near O4$^-$-orientifold is not a stable state.

Behavior of the static potential for $r \to \infty$ can be extracted from an integral representation:

$$\begin{aligned} V_{\mathrm{SO}}(r) &= -\frac{1}{2\sqrt{\pi}} \int \frac{ds}{s^{3/2}} e^{-r^2 s} \sum_{n=1}^{\infty} e^{-8csn} \left(e^{10cs} + 5e^{2cs} + 2e^{6cs} - 2 - 4e^{4cs} - 2e^{8cs} \right) \\ &= -\frac{2}{\sqrt{\pi}} \int \frac{ds}{s^{3/2}} e^{-r^2 s} e^{-cs} \left(e^{4cs} + e^{2cs} + 2 \right) \frac{\sinh^3(cs)}{\sinh(4cs)} . \end{aligned} \tag{5.2}$$

Expand the integrand near $s = 0$, which is a valid expansion for $r \to \infty$,

$$\begin{aligned} V_{\mathrm{SO}}(r) &= -\frac{2}{\sqrt{\pi}} \int \frac{ds}{s^{3/2}} e^{-r^2 s} \left((cs)^2 + \frac{(cs)^3}{2} + \mathcal{O}((cs)^4) \right) \\ &= -\frac{c^2}{r^3} + \mathcal{O}\left(\frac{1}{r^5}\right). \end{aligned} \tag{5.3}$$

One finds that the potential is attractive. Thus, much as in O8$^-$-orientifold, the M2 localized near O4$^-$-orientifold will be attracted toward the orientifold and eventually absorbed into it.

5.3 M2 on O4$^+$ orientifold

Turning next to O4$^+$ orientifold corresponding to USp Chan-Paton gauge bundle of zero-brane partons, from the projection rules, one now keeps

$$\begin{aligned} V_{\mathrm{USp}}(r) = \sum_{m=1}^{\infty} \Bigg(& \left\{ \sqrt{r^2 + 2c(4m-3)} + \sqrt{r^2 + 2c(4m+1)} \right\} + \\ & + \left\{ 2\sqrt{r^2 + 2c(4m-1)} + 4\sqrt{r^2 + 2c(4m-3)} \right\} - \\ & - 2\sqrt{r^2 + 2c(4m)} - 4\sqrt{r^2 + 2c(4m-2)} - 2\sqrt{r^2 + 2c(4m-4)} \Bigg). \end{aligned} \tag{5.4}$$

Again, terms in the first line come from projection to odd modes of Y^1, Y^2, which automatically projects to even modes of Y^9, and those in the second line are from projection to odd modes of Y^3, Y^4 and even modes of $Y^5, \ldots, Y^8$, respectively.

From eq. (5.5), one clearly sees that the static potential for $r \to 0$ is well-defined, as the tachyon mode, ($n = 0$ of eq. (2.9), is projected out completely, as in the O8$^+$-orientifold.

For $r \to \infty$, the static potential can be extracted from the integral representation:

$$\begin{aligned} V_{\mathrm{USp}}(r) &= -\frac{1}{2\sqrt{\pi}} \int \frac{ds}{s^{3/2}} e^{-r^2 s} \sum_{n=1}^{\infty} e^{-8csn} \left(e^{-2cs} + 5e^{6cs} + 2e^{2cs} - 2 - 4e^{4cs} - 2e^{8cs} \right) \\ &= \frac{2}{\sqrt{\pi}} \int \frac{ds}{s^{3/2}} e^{-r^2 s} e^{-3cs} \left(2e^{4cs} + e^{2cs} + 1 \right) \frac{\sinh^3(cs)}{\sinh(4cs)}. \end{aligned} \tag{5.5}$$

For $r \to \infty$, one finds

$$\begin{aligned} V_{\mathrm{USp}}(r) &= \frac{2}{\sqrt{\pi}} \int \frac{ds}{s^{3/2}} e^{-r^2 s} \left((cs)^2 - \frac{3(cs)^3}{4} + \mathcal{O}((cs)^4) + \cdots \right) \\ &= +\frac{c^2}{r^3} + \mathcal{O}\left(\frac{1}{r^5}\right), \end{aligned} \tag{5.6}$$

exhibiting a repulsive long-range force between M2 and O8$^+$-orientifold.

5.4 $\mathbf{T}^5/\mathbb{Z}_2$ orientifolds and twisted sector states

If the covering space $\mathcal{M}_5$ of the orientifold is compact, from the analysis of section 4.4, one would expect that the stability as well as static potential of the M2 configuration becomes modified. The modification arises due to interaction between the M2 and twisted sector of the orientifold.

In so far as the twisted sector is concerned, there is one important difference of the O4-orientifold from the O8-orientifold. It has actually more to do with peculiarity of the twisted sector in O8 orientifold. For example, the twisted sector of the $\mathrm{O8}^-$ orientifold consists only of fermions that are gauge singlets under the Chan-Paton gauge group [14, 13, 15].

In contrast, for $\mathrm{O4}^+$ orientirfold, the twisted sector is described by (dimensional reduction of) chiral multiplets of $d = 6$, $\mathcal{N} = 1$ supersymmetric gauge theory. Thus, the twisted sector modes are localized at the fixed points and do not couple to fields along the orientifold directions, viz. the excitation energy spectrum is again a function of r alone. As such, when summing up the fluctuations, bosonic contribution of the twisted sector is cancelled exactly by the fermionic contribution. Thus, for M2 near O4 orientifold, the twisted sector does not lead to any change to the stability criterion nor to the static potential energy.

5.5 Summary

M2 is a stable, non-BPS configuration when located on $\mathrm{O4}^+$ orientifold, but exhibits a tachyonic instability if located on $\mathrm{O4}^-$ orientifold.

6. Comparison with K-theoretic classification

One would like to see whether the stable configuration of non-BPS M2 on orientifolds analyzed in the previous section can be understood from an independent analysis, for example, K-theory classification. The relevant K-theoretic analysis has been already given in [8, 9] . According to [6, 7], the D-brane charges in Type IIA string theory takes values in $\mathrm{K}^{-1}(X)$, the subgroup of $\mathrm{K}(S^1 \times X)$ consisting of trivial elements when restricted to a point on S^1.

For $\mathbb{Z}_2$ orientifolds, according to the proposal of [9], the D-brane charges of Dq-brane on Op-orientifold is classified by[4]

$$\mathrm{KO}(\mathrm{D}^{p-q}, \partial \mathrm{D}^{p-q}) = \mathrm{KO}^{-(p-q)}(\cdot)\,, \tag{6.1}$$

for Op-orientifold which projects the Chan-Paton gauge bundle of Dp-brane to SO-type, and

$$\mathrm{KSp}(\mathrm{D}^{p-q}, \partial \mathrm{D}^{p-q}) = \mathrm{KSp}^{-(p-q)}(\cdot) = \mathrm{KO}^{-(p-q+4)}(\cdot)\,, \tag{6.2}$$

for Op-orientifold which projects the Chan-Paton gauge bundle of Dp-brane to USp-type, respectively.

[4]We are grateful to K. Hori for helpful discussions on these results.

Applied to the cases of interest, where $p-q=2$ or 6, one finds

$$\begin{aligned} p-q=2 \quad : \quad & \mathrm{KO}^{-2}(\cdot) = \mathbb{Z}_2 && \text{for} \quad \text{SO-projection} \\ & \mathrm{KSp}^{-2}(\cdot) = 0 && \text{for} \quad \text{USp-projection} \end{aligned} \tag{6.3}$$

and

$$\begin{aligned} p-q=6 \quad : \quad & \mathrm{KO}^{-6}(\cdot) = 0 && \text{for} \quad \text{SO-projection} \\ & \mathrm{KSp}^{-6}(\cdot) = \mathbb{Z}_2 && \text{for} \quad \text{USp-projection.} \end{aligned} \tag{6.4}$$

Recall that we have adopted, from M(atrix) theory point of view, a more natural notation for classifying orientifold projection to SO or USp Chan-Paton gauge bundle was in terms of that of zero-brane partons of the M(atrix) theory (see the footnote 3). In the weak coupling limit of the M-theory, the zero-brane parton is identified with the D0-brane of Type IIA string theory and, similarly, M-theory orientifolds with Type IIA orientifolds. One can thus associate, taking into account of the Bott periodicity, the M(atrix) $\mathrm{O8}^{\pm}$ orientifolds with Type IIA O8 orientifolds with projection to USp(SO) Chan-Paton gauge bundles, and M(atrix) $\mathrm{O4}^{\pm}$ orientifolds with Type IIA O4 orientifolds with projection to SO (USp) gauge bundles.

One thus finds that the K-theory classification given in eqs. (6.3), (6.4) matches perfectly with the result we have deduced directly from M(atrix) theory in the previous sections, provided the latter is interpreted in the weak coupling, Type IIA string theory limit of the M-theory.

7. Conclusion

In this paper, in M(atrix) theory, we have presented a few examples of a stable, non-BPS M-brane configuration on an M-orientifold plane. We have proposed M(atrix) theory rules for orientifold projection to the fluctuation spectrum around a given configuaration and hence for stability criterion. We have found that the examples found directly using the rules are in perfect agreement with the predictions based on K-theoretic classification. Our approach is inherently based on M(atrix) theoretic description of BPS branes and orientifolds and is readily applicable to the Type IIB Matrix theory [20] or to the Type I Matrix theory [21].

Moreover, combining the method developed in this paper with recent work of [22], where M(atrix) theory $(p-2)$-brane is constructed as a "vortex" configuration of the effective abelian Higgs model on the worldvolume of p-brane and $\overline{p}$-brane pair, it ought to be possible to explore deeper aspects of stable (BPS or not) branes in M-theory. Further results along this direction will be reported elsewhere.

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